Cincinnati Country Day School

AP Calculus (AB and BC) Summer Packet

Directions: This packet is due on the first day of class of the 2019-2020 school year. Copy each problem onto a separate piece of paper and solve. Show your procedure, not just your answer. If you use a graph you should show a properly labeled sketch of that graph. Expect an assessment on this material during the first week of school. If you are looking for resources to help assist with this packet, you can find some on the following OneNote page: http://bit.ly/1RkArAw

Skill 1: Find the slope between two points.

Calculus is the study of slope, so it's a good idea to review that $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example: The slope between $(-3, 5)$ and $(6, -4)$ is $\dfrac{-4-5}{6-(-3)} = \dfrac{-9}{9} = -1$.

Skill 2: Write the equation of a line.

We do this so often in calculus, that it has become a mantra: *To write the equation of a line, you need two things: **1)** a point and **2)** a slope.* We will always use the point-slope formula for a line:

$$y - y_1 = m(x - x_1)$$

Example: The equation of the line that has slope 2 and goes through the point $(3, -1)$ is $y + 1 = 2(x - 3)$. If for some reason, we needed the equation in slope-intercept form, we would simply solve for y: $y = 2x - 7$.

1. Determine the equations of the following lines. Give your answer in *point-slope* form.
   a. the line through (-1,3) and (2,-4);

   b. the line through (-1; 2) and perpendicular to the line 2x- 3y + 5 = 0;

   c. the line through (2, 3) and the midpoint of the line segment from (-1, 4) to (3, 2).

Skill 3: Evaluate piecewise functions.

You won't need to know how to graph piecewise functions, but you will need to know how to plug numbers into a piecewise function. Whichever condition the number satisfies, use that formula to plug the number into.

Example: If $f(x) = \begin{cases} x^2+2 & \text{if } x > 1 \\ 4x-1 & \text{if } x \leq 1 \end{cases}$, find $f(3)$, $f(1)$, and $f(0)$.

Since $3 > 1$, we use the top equation: $f(3) = 3^2 + 2 = 11$.

For $f(1)$, we note that $1 \leq 1$, so we use the bottom equation: $f(1) = 4(1) - 1 = 3$.

Also, since $0 \leq 1$, so we use the bottom equation: $f(0) = 4(0) - 1 = -1$.

2. Given $g(x) = \begin{cases} 3x-1, & \text{if } x < -2 \\ x^2+4, & \text{if } -2 \leq x < 4 \\ -2x+1, & \text{if } x > 4 \end{cases}$, evaluate the following:

   a. $g(3)$

   b. $g(4)$

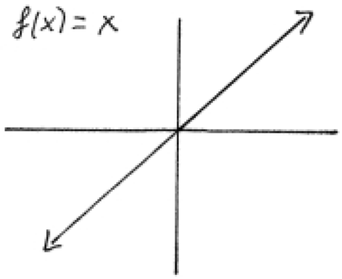
   c. $g(-5)$

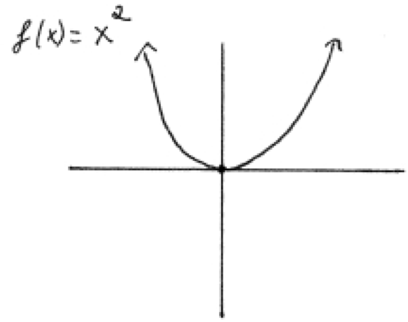
   d. $\dfrac{g(x+h)-g(x)}{h}$, given $x < -2$

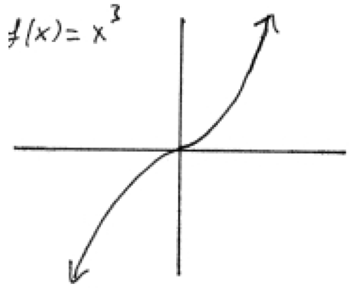
Skill 4: Identify the parent graphs.

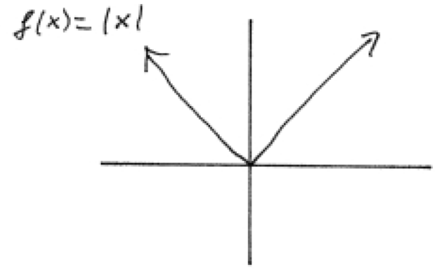
The two most important graphs you should know are $y = e^x$ and $y = \ln x$. There are 10 parent graphs in total and they are on the back of this sheet.

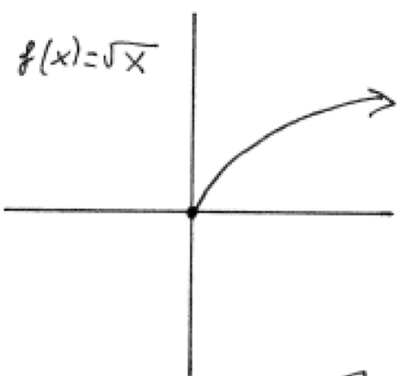
The Big Ten

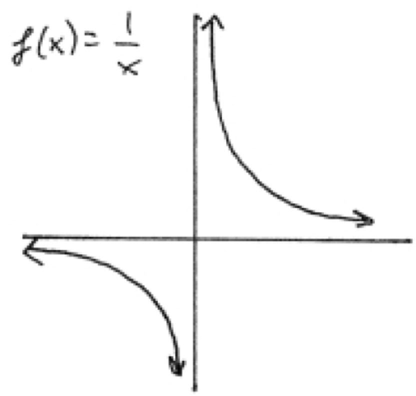


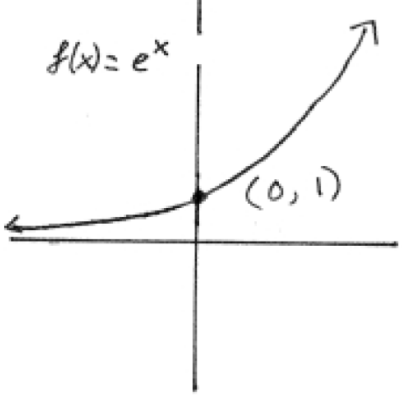


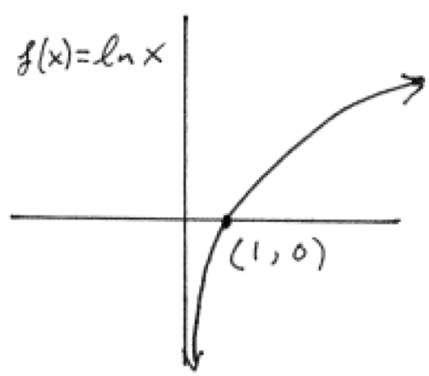


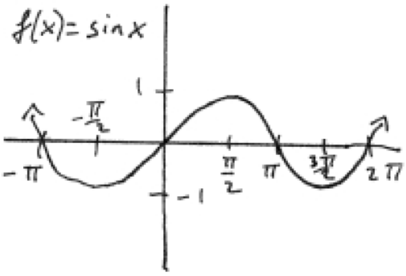


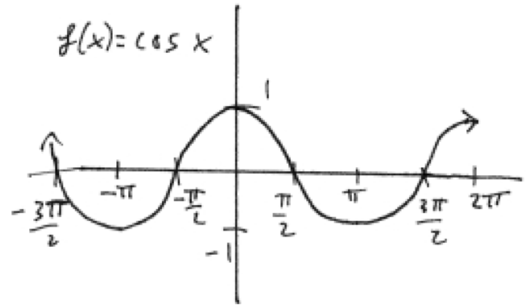












3. Use transformations to sketch the following graphs:

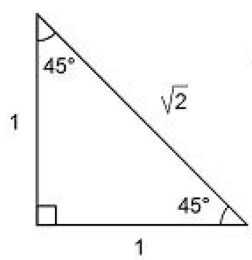
   a. $f(x) = \ln(x-2)+4$

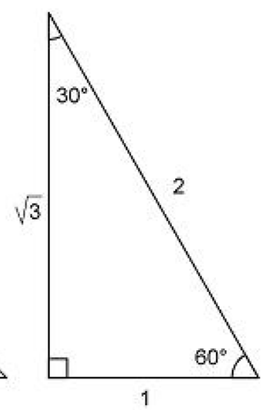
   b. $f(x) = e^{-x} - 3$

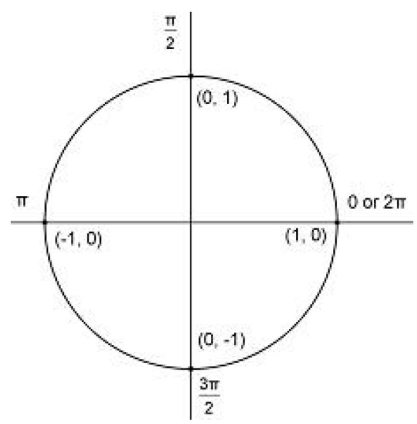
   c. $f(x) = -\sqrt{x+2}+5$

Skill 5: Find basic trig values using the reference triangles and the unit circle.

You should know the 2 reference triangles and the unit circle like the back of your hand. Remember on the unit circle, the cosine is the x-coordinate and the sine is the y-coordinate (it's alphabetical).

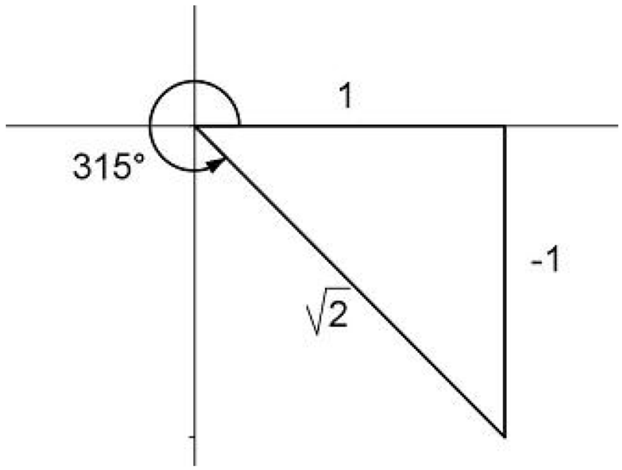






In calculus, we will **ALWAYS** use *radians*, so know the basic conversions: $\frac{\pi}{6} = 30°$, $\frac{\pi}{4} = 45°$, $\frac{\pi}{3} = 60°$.
If you have an angle greater than $\frac{\pi}{2}$, then place the reference triangle in the appropriate quadrant on the coordinate plane remembering that up/right means positive and down/left means negative.

Example: To find $\sin\frac{7\pi}{4}$, we first realize that the angle is in the 4th quadrant since $\frac{7\pi}{4}$ is just short of $\frac{8\pi}{4} = 2\pi$. We draw the reference triangle with a positive adjacent side since it is to the right of the origin and a negative opposite side since it is below the origin. Of course, the hypotenuse is always positive. From the triangle, we see that $\sin\frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$



4. Without a calculator, evaluate the following.

(a) cos 210°

(b) $\sin\frac{5\pi}{4}$

(c) $\tan^{-1}(-1)$

(d) $\sin^{-1}(-1)$

(e) $\cos\frac{9\pi}{4}$

(f) $\sin^{-1}\frac{\sqrt{3}}{2}$

(g) $\tan\frac{7\pi}{6}$

(h) $\cos^{-1}(-1)$

(i) $\sin\frac{\pi}{6}$

(j) $\tan\frac{7\pi}{6}$

(k) $\cos 0$

(l) $\cos\frac{\pi}{4}$

(m) $\csc\left(\frac{-5\pi}{6}\right)$

(n) $\sec\pi$

(o) $\cot\left(\frac{-\pi}{2}\right)$

(p) $\tan\frac{\pi}{2}$

(q) $\sin\frac{5\pi}{6}$

(r) $\cot\frac{2\pi}{3}$

(s) $\sin\frac{\pi}{2}$

(t) $\sec\frac{3\pi}{4}$

(u) $\csc\pi$

(v) $\sec\frac{11\pi}{6}$

(w) $\cot\frac{4\pi}{3}$

(x) $\cos^{-1}\frac{\sqrt{3}}{2}$

(y) $\cot^{-1}(-1)$

(z) $\tan^{-1}(-1)$

(aa) $\sin^{-1}\left(-\frac{1}{2}\right)$

(bb) $\sin\left(\csc^{-1}(-2)\right)$

(cc) $\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)$

Skill 6: Solve simple trig equations using the unit circle.

Example: To solve $\sin x = 0$, we look for the angles on the unit circle where the y-coordinate is 0. This is the left and right points, so the solutions are the angles $x = 0, \pi$.

5. Solve the following trigonometric equations on the interval $0 \le x \le 2\pi$

   a. $2\sin x = 1$

   b. $2\cos^2 x - 3\cos x + 1 = 0$

   c. $3\sin^2 x = \cos^2 x$

Skill 7: Rewrite common expressions using exponents.

In calculus, it is usually advantageous to write roots and fractions as x to a power:

Examples: $\sqrt{x} = x^{\frac{1}{2}}$ $\quad \sqrt[3]{x} = x^{\frac{1}{3}}$ $\quad \frac{1}{x} = x^{-1}$ $\quad \frac{1}{x^2} = x^{-2}$ $\quad \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

6. Rewrite the following expressions using exponents.

   a. $\dfrac{3}{\sqrt{x-1}}$ b. $\dfrac{4x}{(x^2-3)^5}$ c. $\dfrac{-5}{\sqrt[3]{(x-1)^2}}$

Skill 8: Evaluate logarithms.

The *only* logarithm that is used in calculus is the natural logarithm $y = \ln x$. Really the only thing you need to know is that ln and e cancel each other out.

Examples: $\ln e^5 = 5$ $\quad \ln \frac{1}{e^2} = \ln e^{-2} = -2$ $\quad \ln e = \ln e^1 = 1$ $\quad \ln 1 = \ln e^0 = 0$

Skill 9: Solve simple logarithmic equations.

To solve a logarithmic equation we write the equation in exponential form.

Example: $\ln x = 6 \Rightarrow e^6 = x$

7. Solve the following logarithmic equations.

a. $\ln x + 3 = 5$ b. $\ln(x+3) = 5$ c. $\ln(x-1)^2 = 8$

Skill 10: Calculate limits.

1. Find the limit:
1 pts.

$$\lim_{x \to -3} \frac{x^2 - 9}{x^2 + x - 6}$$

- A. 1
- B. 3/2
- C. 6/5
- D. -9
- E. 3

2. Find the limit:
1 pts.

$$\lim_{x \to -\infty} \frac{8x^4 + 2x - 1}{5 + 7x - 2x^4}$$

- A. 8
- B. 8/5
- C. 4
- D. -4
- E. 1/2

3. Find the limit:
1 pts.

$$\lim_{x \to \infty} \frac{(2x+7)^2}{(3x-1)(4-x)}$$

- A. 4/3
- B. -4/3
- C. 2/3
- D. -2/3
- E. 1/3

4. Find the limit:
1 pts.

$$\lim_{x \to 4^+} \frac{3}{4 - x}$$

- A. Positive Infinity
- B. Negative Infinity
- C. 0
- D. 3/4
- E. -3

5. Find the limit:
1 pts.

$$\lim_{x \to 7^-} \frac{2}{(x-7)^2}$$

- A. Positive Infinity
- B. Negative Infinity
- C. 2/49
- D. 0

11. Find the limit:
1 pts.

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$$

- A. 2
- B. 4
- C. 1/2
- D. 1/4
- E. 1

12. Find the limit:
1 pts.

$$\lim_{x \to \infty} \frac{5 - 3x^5}{2x^5 + 7x - 1}$$

- A. 5/2
- B. -3/2
- C. -5
- D. 3
- E. Infinity

13. Find the limit:
1 pts.

$$\lim_{x \to \infty} \frac{(5x-1)(x+2)(3x+2)}{(2x-3)^3}$$

- A. 5/2
- B. 15/2
- C. 15/8

14. Find:
1 pts.

$$\lim_{x \to 5} \frac{x - 5}{x^2 - 3x - 10}$$

- A. 1
- B. 1/2
- C. 7
- D. 1/7

Skill 11: Find limits using a graph.

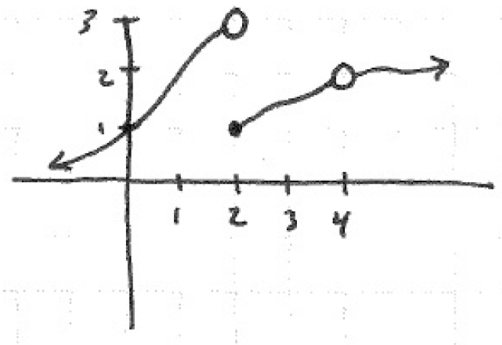
1. Consider the graph below. Which of the following statements are true?

1 pts.

I. $\lim_{x\to 4} f(x) = \lim_{x\to 2^+} f(x)$

II. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x)$

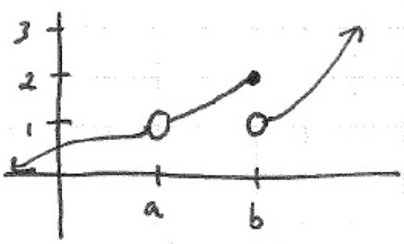
III. $\lim_{x\to 2^-} f(x) = f(2)$



- A. II only
- B. III only
- C. I and II
- D. I and III
- E. II and III

2. Consider the graph below. Which statement is false?

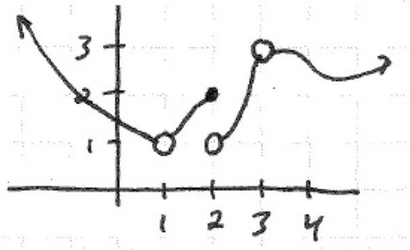
1 pts.



- A. $f(a)$ does not exist
- B. $\lim_{x\to a^-} f(x) = 1$
- C. $\lim_{x\to a} f(x)$ does not exist
- D. $\lim_{x\to b} f(x)$ does not exist
- E. $f(b) = 2$

3. .
1 pts.

Referring to the graph below, if $\lim_{x \to c} f(x) = 1$, then what must c equal?



- A. 1
- B. 2
- C. 3
- D. 4
- E. none of the above

<u>**Skill 12**</u>: Identify asymptotes of functions.

4. .
1 pts.

What kind of asymptote does $\lim_{x \to 5} f(x) = -\infty$ describe?

- A. Horizontal
- B. Vertical

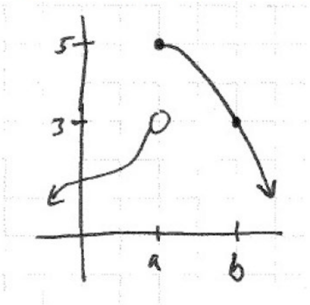
6. Which function has a horizontal asymptote of y = 3? (THERE MAY BE MORE THAN ONE CORRECT ANSWER - SELECT ALL THAT APPLY) (Choose all that Apply)
1 pts.

- A. $y = 3x$
- B. $y = e^x + 3$
- C. $y = \dfrac{-3x^2 + 5x - 1}{6 - x^2}$
- D. $y = \dfrac{x+3}{x+1}$
- E. $y = \dfrac{1}{x-3}$

Skill 13: Determine the continuity of a function.

1. Consider the graph below. Which statement is false?

1 pts.



- A. $\lim_{x \to a} f(x)$ does not exist
- B. $\lim_{x \to b} f(x) = 3$
- C. f is defined at $x = a$ (This means that $f(a)$ exists)
- D. f is continuious at $x = a$
- E. f is continuious at $x = b$

2. At what value(s) of x is the function below discontinuous?

1 pts.

$$f(x) = \frac{(x+1)^2(x-2)}{(x+1)(x-3)}$$

- A. -1 only
- B. 3 only
- C. -1 and 3 only
- D. -1, 2, and 3
- E. f is continuous for all values of x

3. .

1 pts.

For what value of c is $f(x) = \begin{cases} 3x - 7 & \text{if } x \le 1 \\ 2x + c & \text{if } x > 1 \end{cases}$ continuous?

- A. -7
- B. -6
- C. 1
- D. 3
- E. 8