Cincinnati Country Day School

AP Calculus BC Summer Packet

Directions: The AP Calculus BC curriculum consists of 10 total units, where most of the first 3 were covered in Pre-AP Calculus BC. This packet, which is due on the first day of class of the 2021-2022 school year, is a review of these first 3 units. Copy each problem onto a separate piece of paper and solve. Show your procedure, not just your answer. If you use a graph, you should show a properly labeled sketch of that graph.

Skill 1: Calculate limits.

1. Find the limit:
1 pts.

$$\lim_{x\to -3}\frac{x^2-9}{x^2+x-6}$$

- A. 1
- B. 3/2
- C. 6/5
- D. -9
- E. 3

2. Find the limit:
1 pts.

$$\lim_{x\to -\infty}\frac{8x^4+2x-1}{5+7x-2x^4}$$

- A. 8
- B. 8/5
- C. 4
- D. -4
- E. 1/2

3. Find the limit:
1 pts.

$$\lim_{x\to \infty}\frac{(2x+7)^2}{(3x-1)(4-x)}$$

- A. 4/3
- B. -4/3
- C. 2/3
- D. -2/3
- E. 1/3

4. Find the limit:
1 pts.

$$\lim_{x\to 4^+}\frac{3}{4-x}$$

- A. Positive Infinity
- B. Negative Infinity
- C. 0
- D. 3/4
- E. -3

5. Find the limit:
1 pts.

$$\lim_{x\to 7^-}\frac{2}{(x-7)^2}$$

- A. Positive Infinity
- B. Negative Infinity
- C. 2/49
- D. 0

11. Find the limit:
1 pts.

$$\lim_{x\to 2}\frac{x-2}{x^2-4}$$

- A. 2
- B. 4
- C. 1/2
- D. 1/4
- E. 1

12. Find the limit:
1 pts.

$$\lim_{x\to \infty}\frac{5-3x^5}{2x^5+7x-1}$$

- A. 5/2
- B. -3/2
- C. -5
- D. 3
- E. Infinity

13. Find the limit:
1 pts.

$$\lim_{x\to \infty}\frac{(5x-1)(x+2)(3x+2)}{(2x-3)^3}$$

- A. 5/2
- B. 15/2
- C. 15/8

14. Find:
1 pts.

$$\lim_{x\to 5}\frac{x-5}{x^2-3x-10}$$

- A. 1
- B. 1/2
- C. 7
- D. 1/7

Skill 2: Find limits using a graph.

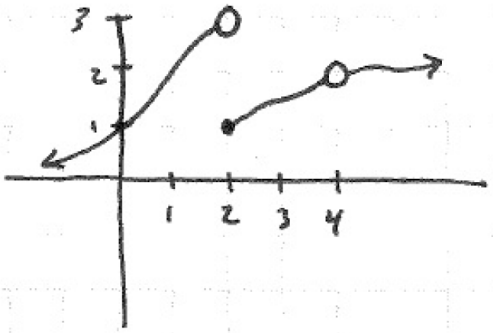
1. Consider the graph below. Which of the following statements are true?

1 pts.

I. $\lim_{x\to 4} f(x) = \lim_{x\to 2^+} f(x)$

II. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x)$

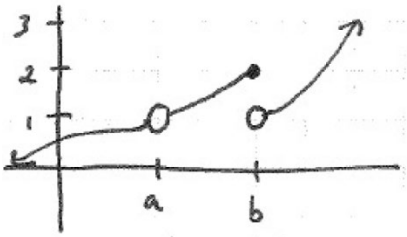
III. $\lim_{x\to 2^-} f(x) = f(2)$



- A. II only
- B. III only
- C. I and II
- D. I and III
- E. II and III

2. Consider the graph below. Which statement is false?

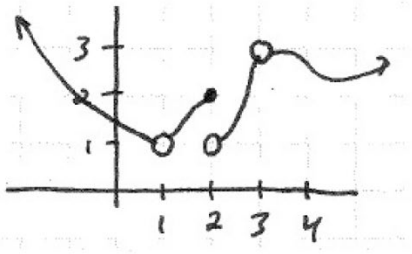
1 pts.



- A. $f(a)$ does not exist
- B. $\lim_{x\to a^-} f(x) = 1$
- C. $\lim_{x\to a} f(x)$ does not exist
- D. $\lim_{x\to b} f(x)$ does not exist
- E. $f(b) = 2$

3. .
1 pts.

Referring to the graph below, if $\lim_{x \to c} f(x) = 1$, then what must c equal?



- A. 1
- B. 2
- C. 3
- D. 4
- E. none of the above

Skill 3: Identify asymptotes of functions.

4. .
1 pts.

What kind of asymptote does $\lim_{x \to 5} f(x) = -\infty$ describe?

- A. Horizontal
- B. Vertical

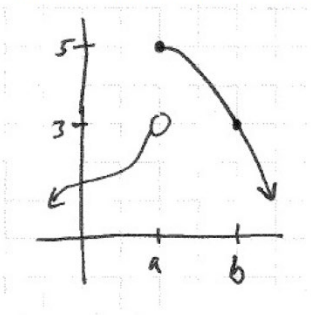
6. Which function has a horizontal asymptote of y = 3? (THERE MAY BE MORE THAN ONE CORRECT ANSWER - SELECT ALL THAT APPLY) (Choose all that Apply)
1 pts.

- A. $y = 3x$
- B. $y = e^x + 3$
- C. $y = \dfrac{-3x^2 + 5x - 1}{6 - x^2}$
- D. $y = \dfrac{x + 3}{x + 1}$
- E. $y = \dfrac{1}{x - 3}$

Skill 4: Determine the continuity of a function.

1. Consider the graph below. Which statement is false?

1 pts.



- A. $\lim_{x \to a} f(x)$ does not exist
- B. $\lim_{x \to b} f(x) = 3$
- C. f is defined at $x = a$ (This means that $f(a)$ exists)
- D. f is continuious at $x = a$
- E. f is continuious at $x = b$

2. At what value(s) of x is the function below discontinuous?

1 pts.

$$f(x) = \frac{(x+1)^2(x-2)}{(x+1)(x-3)}$$

- A. -1 only
- B. 3 only
- C. -1 and 3 only
- D. -1, 2, and 3
- E. f is continuous for all values of x

3. .

1 pts.

For what value of c is $f(x) = \begin{cases} 3x - 7 & \text{if } x \leq 1 \\ 2x + c & \text{if } x > 1 \end{cases}$ continuous?

- A. -7
- B. -6
- C. 1
- D. 3
- E. 8

Skill 5: Evaluate derivatives using basic rules.

1.

Given the function $f(x) = 2\sqrt[5]{x^6}$, find $f'(x)$. Express your answer in radical form without using negative exponents, simplifying all fractions.

2.

For the following equation, find $f'(x)$.

$$f(x) = 9x^4 - x^3 - 2$$

3.

For the following equation, evaluate $f'(-1)$.

$$f(x) = -4x^5 + x^3 + x^2$$

4.

Given $f(x) = 2x^2 - x$, find the equation of the tangent line of f at the point where $x = -3$.

Skill 6: Evaluate derivatives using the product and quotient rules.

1.

Given the function $f(x) = 3x^2 - x^2 \cos x$, find $f'(x)$ in any form.

2.

Given the function $f(x) = \sqrt{25x} \sin x$, find $f'(x)$ in any form.

3.

Given the function $f(x) = \frac{3x^2}{4x^2+3}$, find $f'(x)$ in *simplified* form.

4.

Given the function $y = \frac{5-x^3}{1-x}$, find $\frac{dy}{dx}$ in *simplified* form.

Skill 7: Evaluate derivatives using the chain rule.

1.

Given the function $y = 4(x^2 + 9)^{\frac{3}{2}}$, find $\frac{dy}{dx}$

2.

Given the function $f(x) = -\sqrt{\cos x}$, find $f'(x)$.

3.

Given the function $f(x) = 3\cos\left[(2x^2 + 6)^5\right]$, find $f'(x)$.

4.

Given $y = 3\csc(2x)$, find $\frac{dy}{dx}$.

5.

Given the function $y = \sqrt[3]{\frac{4x^3}{5+5x}}$, find $\frac{dy}{dx}$

6.

Given the function $y = \cos(2x^3)\sin^4(x)$, find $\frac{dy}{dx}$

Skill 8: Use implicit differentiation to find derivatives.

1. If $-y^3 - 4y^2 + x^3 = -4y$ then find $\frac{dy}{dx}$ in terms of x and y.

2. If $2x^3 + 4xy = -y^3 + 5$ then find $\frac{dy}{dx}$ in terms of x and y.

3. Given $\sin(x + y) = x^3$, find $\frac{dy}{dx}$ in terms of x and y.

4. If $-4x + y^2 - xy = 0$ then find the equations of all tangent lines to the curve when $x = 2$.